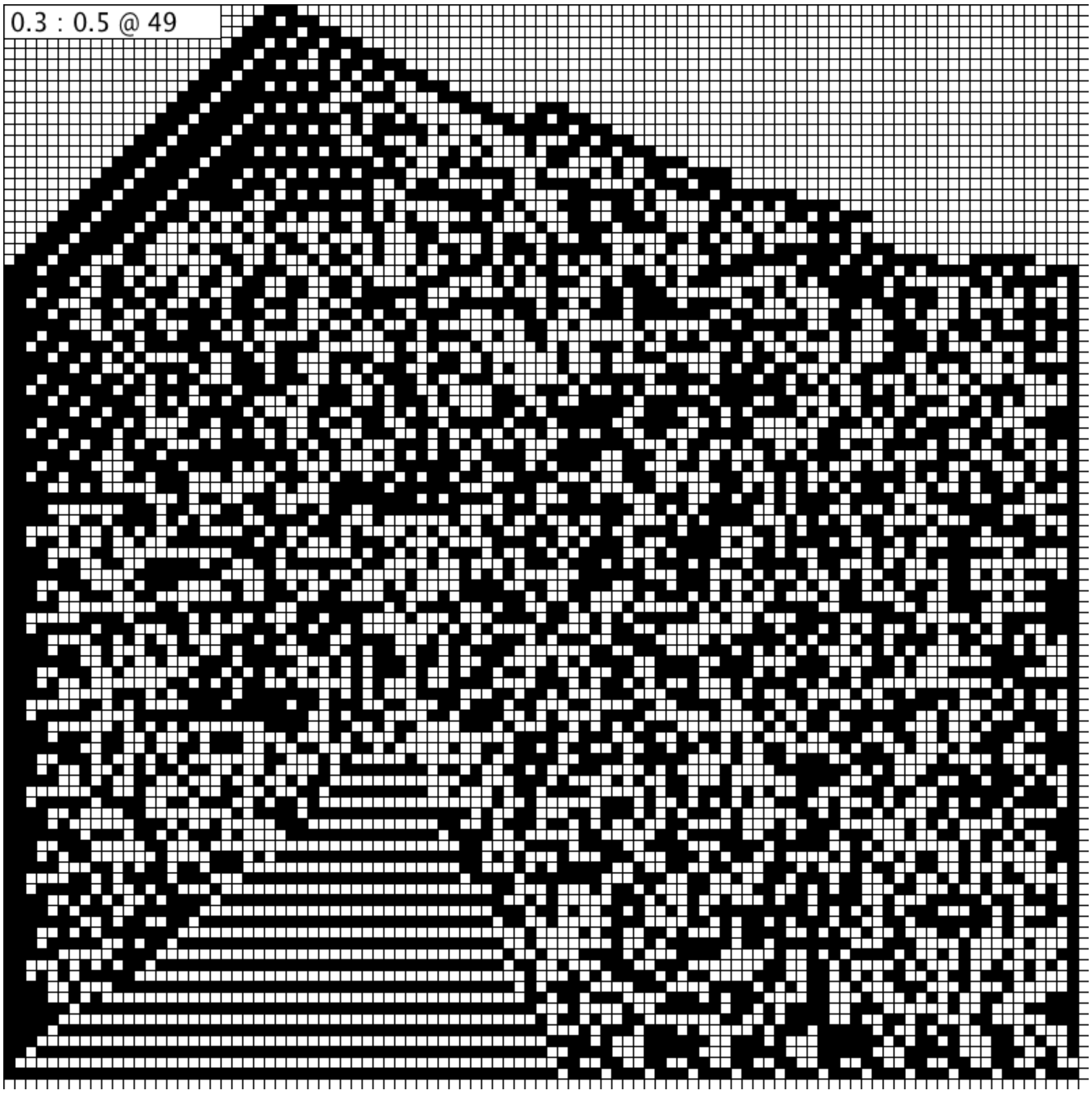
COMPLEX BEAHIVOR IN SIMPLE SYSTEMS



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INTRODUCTION

The goal of this paper is to answer a question concerning a class of very simple systems. 'What are the characteristics of system-rules that will produce complex behavior?" This is by no means a novel question and others have pursued it in the past, with varying degrees of success (Wolfram '03, Kauffman '93, Langton '90). The motivation for this inquiry is to ask the question in a particular way and of a particular type of system that has not yet been done. Wolfram extensively maps the space of one-dimensional cellular automaton. His mapping is very detailed but does not attempt to characterize generally, those rules that have been demonstrated to produce complex behavior (i.e. rule 30, rule 110). Langton explores a class of cellular automaton by observing behavior for different values of a parameter 'lambda'. This exploration led him to coin the term 'the edge of chaos', obviously a very influential result, however Langton did not find a meaningful correlation between his parameter and complexity in binary state systems. Langton's approach has inspired my own, however it is the hypothesis of this paper that finding a parameter that correlates strongly with complexity in binary-state systems is of primary importance. Consider the following notion, supplied by John Archibald Wheeler '90:

It is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer. It from bit. Otherwise put, every 'it'—every particle, every field of force, even the space-time continuum itself—derives its function, its meaning, its very existence entirely—even if in some contexts indirectly—from the apparatus-elicited answers to yes-or-no questions, binary choices, bits.

This notion implies that to understand how complexity arises in the first instance, is to understand the conditions for complexity in binary-state systems. Kauffman is able to find meaningful correlations for systems with five, or greater possible states. But for such systems, a level of complexity is *already assumed.* In a binary system discrete objects can 'emerge' and take many more than just two states. To be precise an emergent object of size *n* can potentially inhabit up to 2^n states. But the emergence of discrete objects and structures that can inhabit many states *is a complex behavior itself*. Starting with five or greater states doesn't get to the root of the problem, rather it can only explain things about systems that already have a degree of complexity built in.

Continuing in the vein of this train of thought, the current paper only examines systems with the simplest possible heterogeneous initial condition – a single element turned on. Under this initial condition, the famed *Conway's Game of Life,* would become homogenous after one iteration – thereby locating it as a Wolfram 'Class 1' system – the simplest possible class. We know however, the *CGOL* is capable of universal computation and exhibits incredibly complex behavior under many initial conditions. It is the hypothesis of this paper that starting with anything more

complex than the simplest possible initial condition is, again, not addressing the prior issue. To re-state the central question of this paper;
"What types of laws will develop complexity in a binary-state system, from the simplest starting point"?

METHODOLOGY

Defining the System

The system is a two-dimensional cellular automaton (CA), where each element can inhabit only two states. Each element considers the adjacent eight elements as neighbors (the Moore neighborhood). The system contains 1000 elements and takes the toroidal arrangement. The system has 2^1000 possible configurations and each neighborhood has 2^9 (512) possible configurations. A given rule will have 512 component sub-rules. Each is defined by mapping each possible neighborhood configuration – here considered the antecedent – to a consequent state for the focal cell (either on or off). Given that there are 512 possible antecedents, each with 2 possible consequents – the aggregate rule-space for our system comprises 2^512 (1.34078079 × 10^154) rules – each comprising 512 component transition rules.

The Parameters

The parameters that were tested for correlation with complex behavior are as follows. These parameters can be thought of as properties of each of the 2^512 possible rules of the system.

Probability of Change (PC): The number of component rules (of a rule) where the focal element's state is differs in the antecedent and the consequent divided by the total number of transition rules.

Change Ratio (CR): The number of transitions from off to on, divided by the number of transitions from on to off.

The Structure of the Experiment

Parameter values ranging from 0.1 to 0.9, in 0.1 increments were tested for. That is to say that in each run of the experiment, 81 rules were generated and executed on the system described above – from a single 'turned on' element. The experiment was run seven times, so 567 rules were observed. Clearly this represents an incredibly miniscule subset of the 2^512 possible rules of the system. Rules were generated dynamically and randomly to conform to the specification of the parameters – and uniqueness of rules was enforced. The output of the system under each rule was subjectively interpreted in terms of the Wolfram Classes – and accorded a score between 1 and 4 inclusive. The Wolfram Classes are defined below:

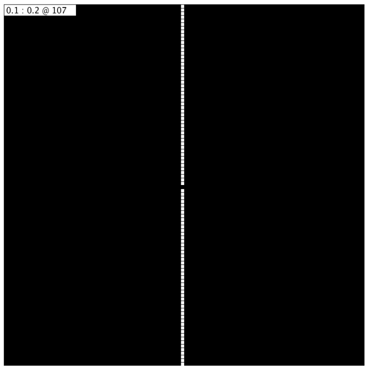
Class 1: Nearly all-initial patterns evolve quickly into a stable, homogeneous state. Any randomness in the initial pattern disappears.
Class 2: Nearly all-initial patterns evolve quickly into stable or oscillating structures. Some of the randomness in the initial pattern may filter out, but some remains. Local changes to the initial pattern tend to remain local.
Class 3: Nearly all-initial patterns evolve in a pseudo-random or chaotic manner. Any stable structures that appear are quickly destroyed by the surrounding noise. Local changes to the initial pattern tend to spread indefinitely.
Class 4: Nearly all-initial patterns evolve into structures that interact in complex and interesting ways. Class 2 type stable or oscillating structures may be the eventual outcome, but the number of steps required to reach this state may be very large, even when the initial pattern is relatively simple. Local changes to the initial pattern may spread indefinitely. Wolfram has conjectured that many, if not all class 4 cellular automata are capable of universal computation.

Analysis

The analysis is simple but revealing. Since the Wolfram classification values non-Class 4 systems as of equal complexity, the key metric is 'how many class 4 systems were observed under parameter value *x'?* Extensive visual data has been supplied to support the numerical analysis.

RESULTS

These images are of the system under various rules. The parameter characterization of the rule is indicated in the top left corner in the format PC : CR @ iteration. Some snap shots are of systems in their resting state – be it static, chaotic or homeostatic. Some snap shots are of systems in development stages. Sadly, what is not captured by these images is the way in which the various systems move, which plays a key role in determining how a given system will be classified.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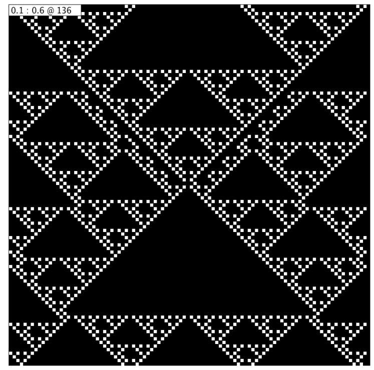
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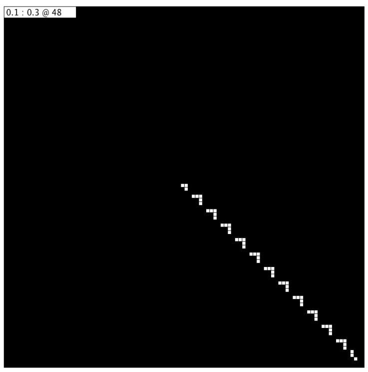
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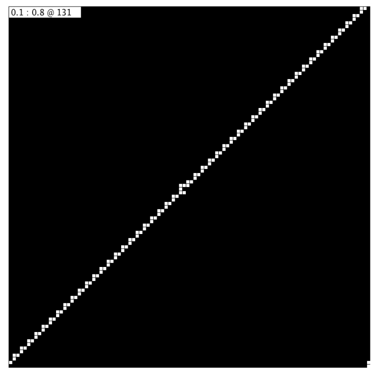
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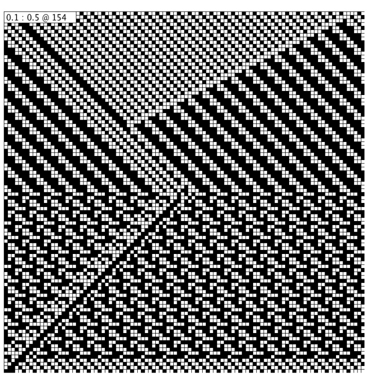
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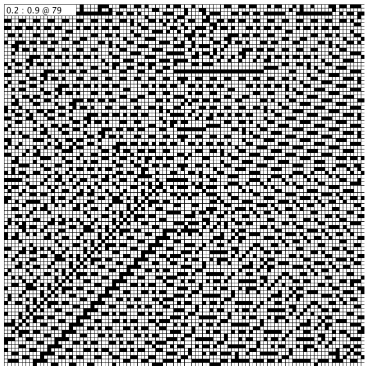
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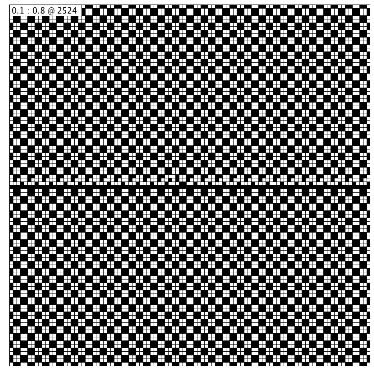
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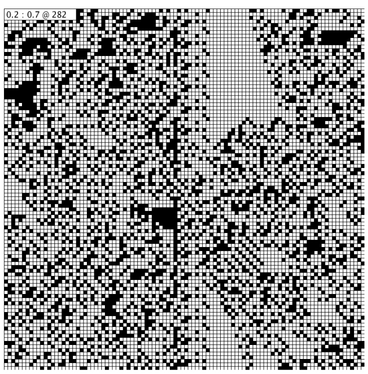
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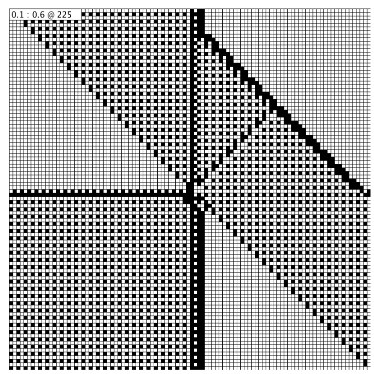
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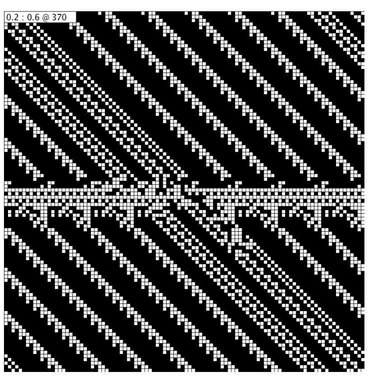
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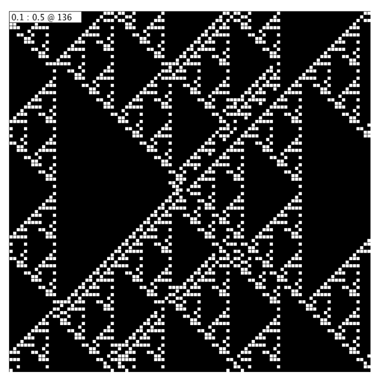
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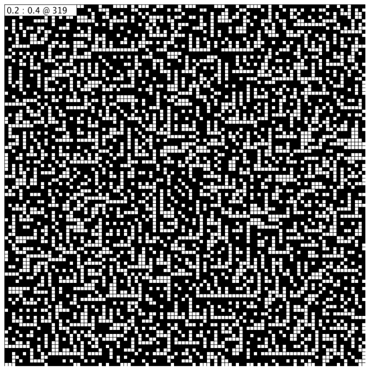
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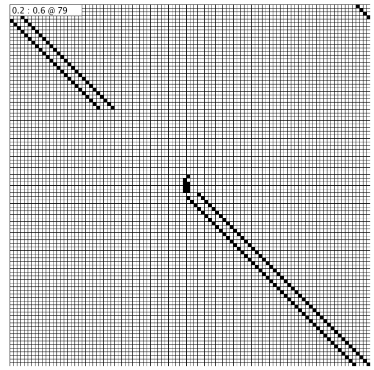
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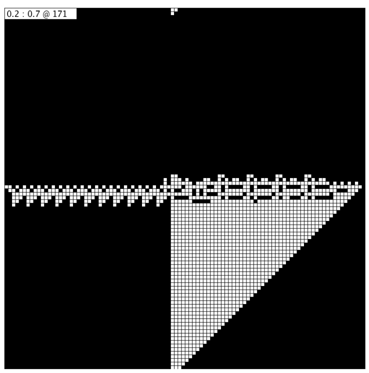
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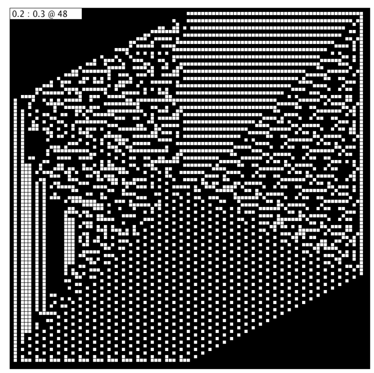
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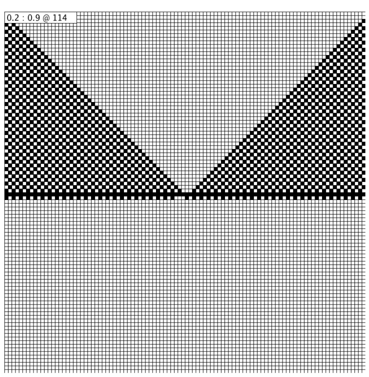
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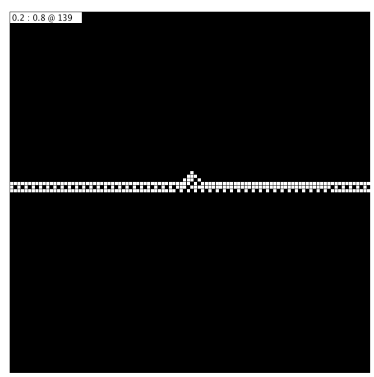
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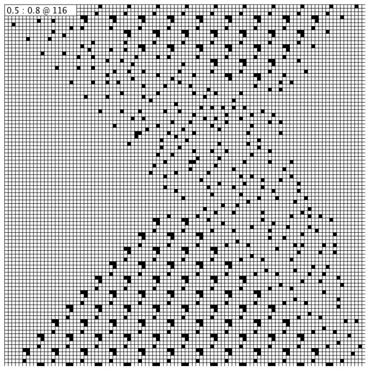
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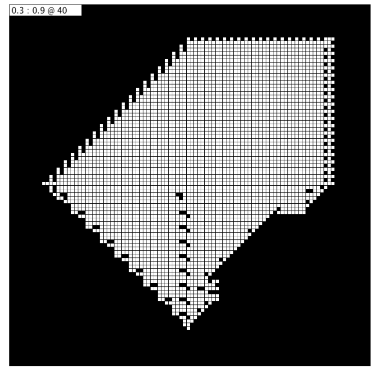
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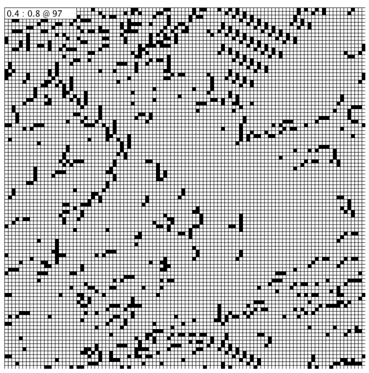
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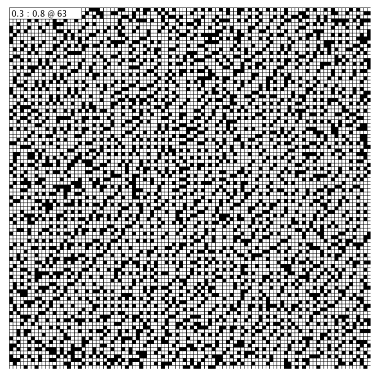
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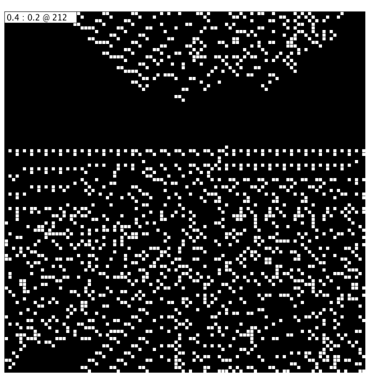
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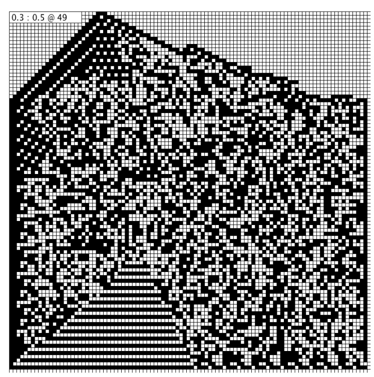
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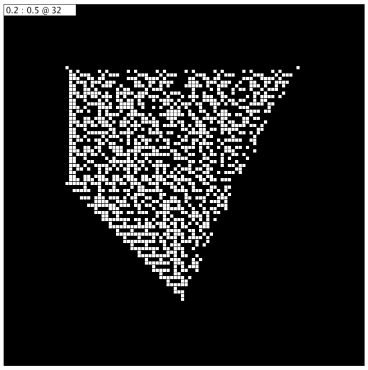
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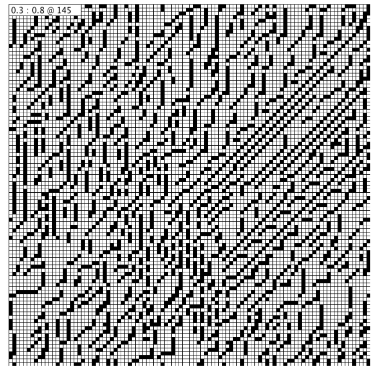
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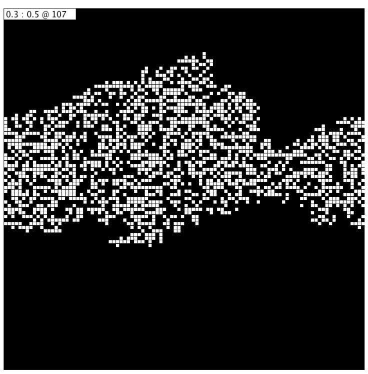
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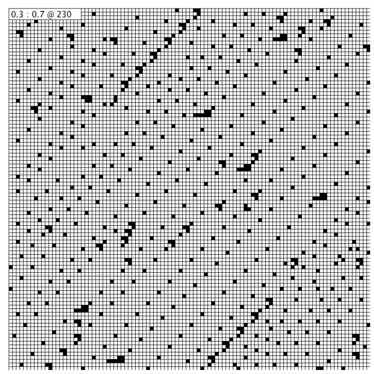
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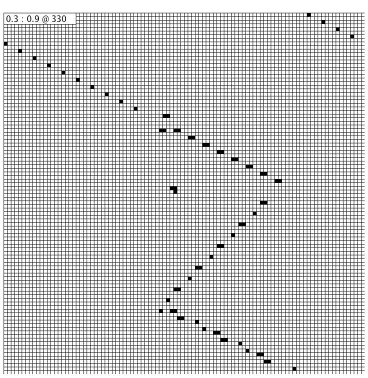
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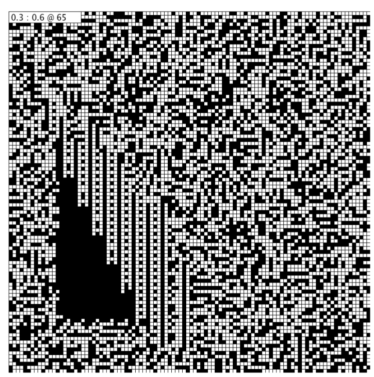
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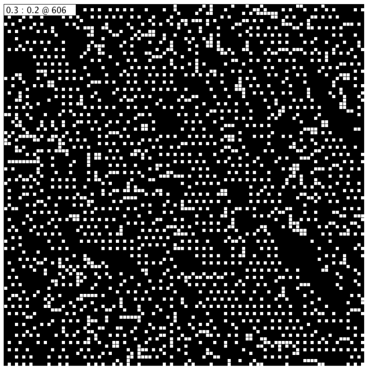
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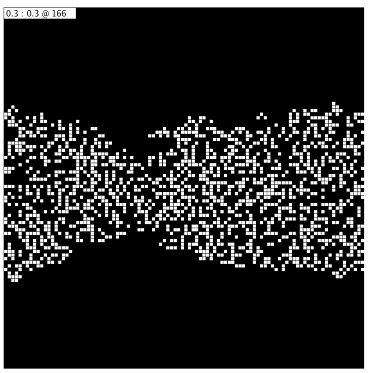
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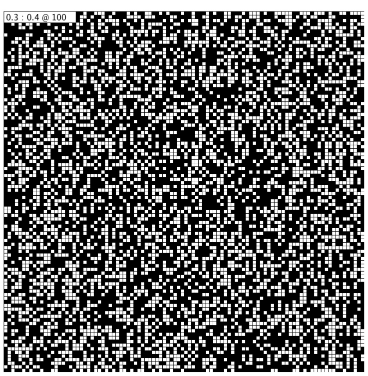
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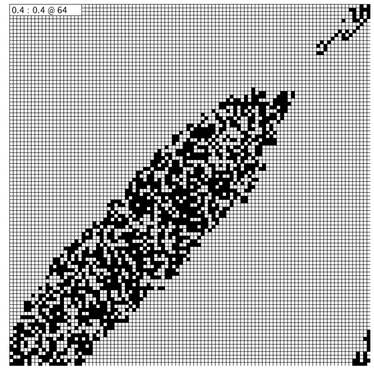
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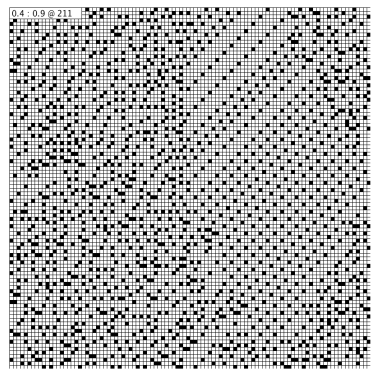
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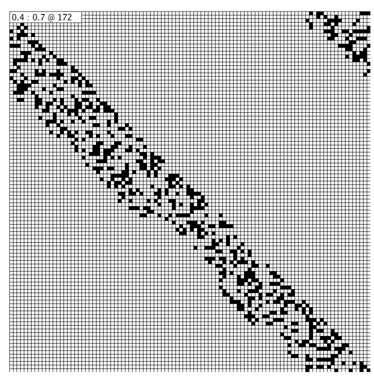
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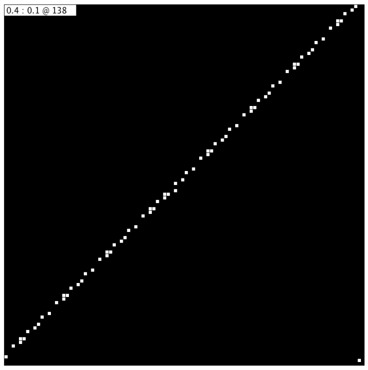
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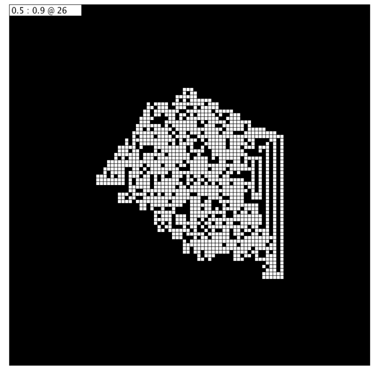
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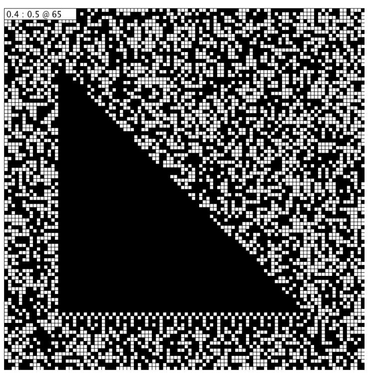
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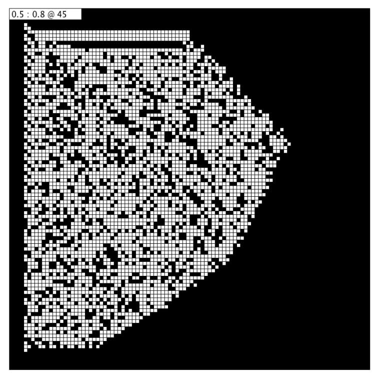
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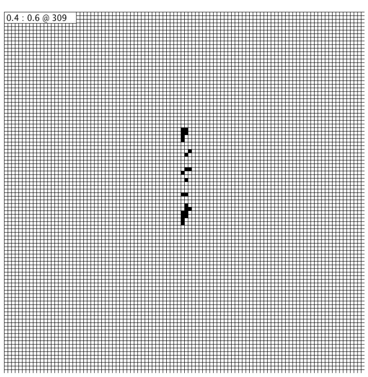
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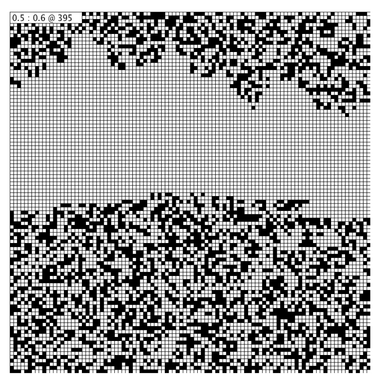
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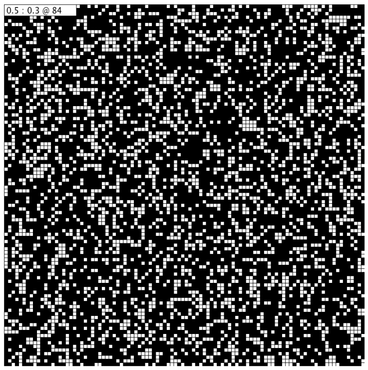
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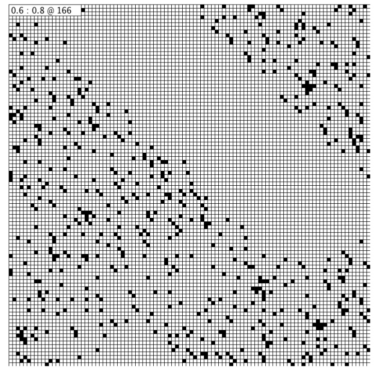
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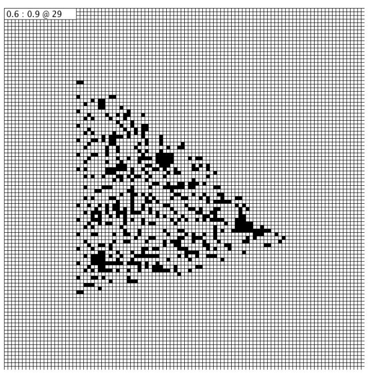
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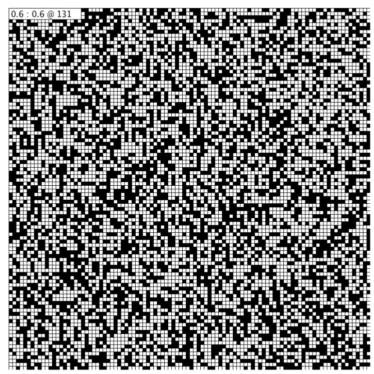
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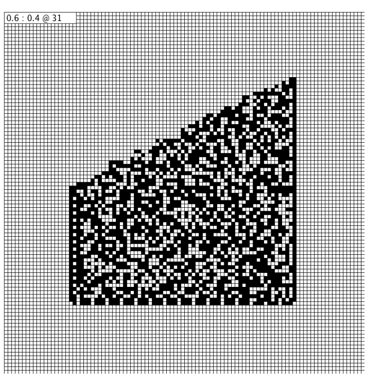
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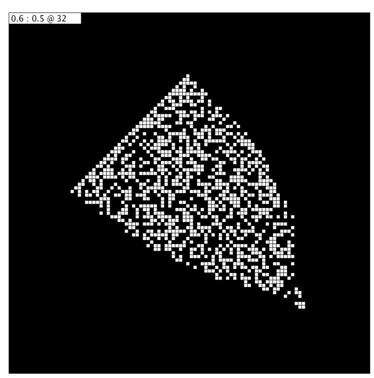
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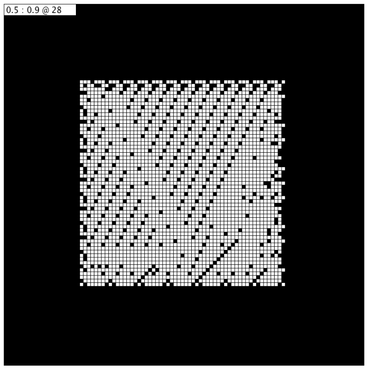
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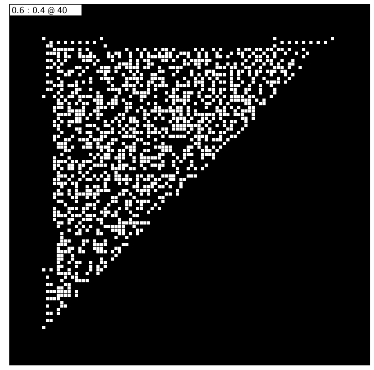
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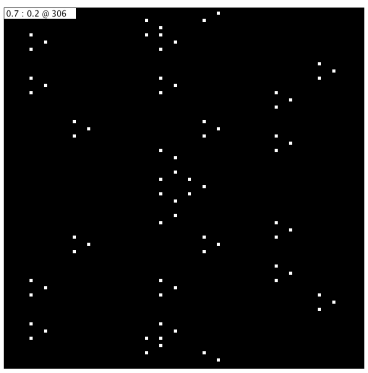
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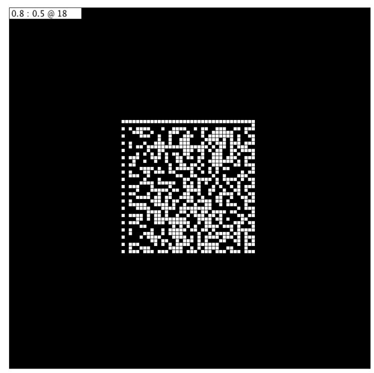
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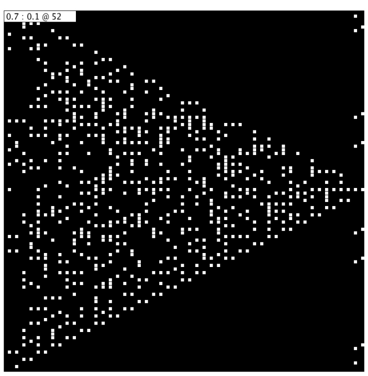
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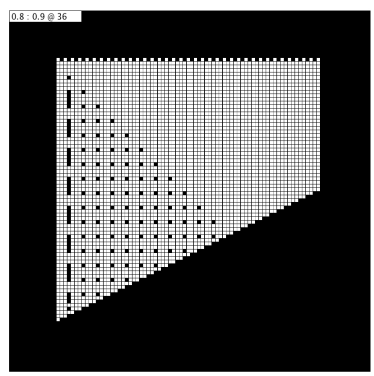
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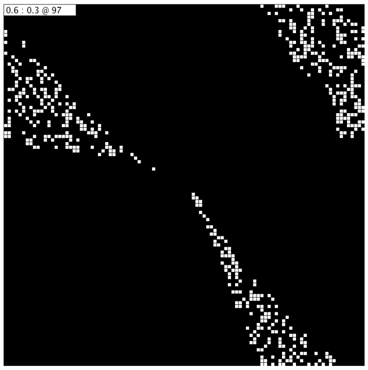
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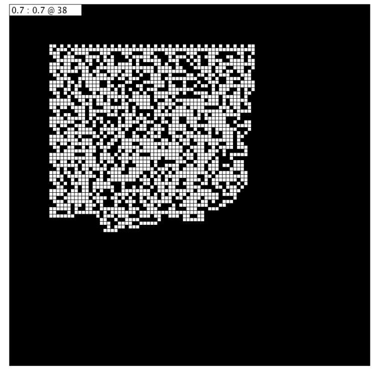
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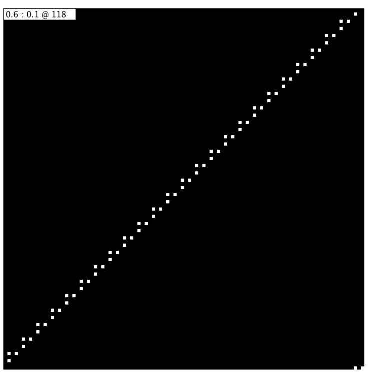
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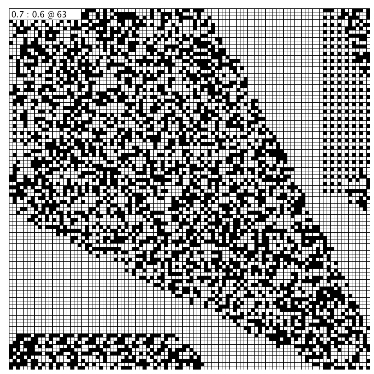
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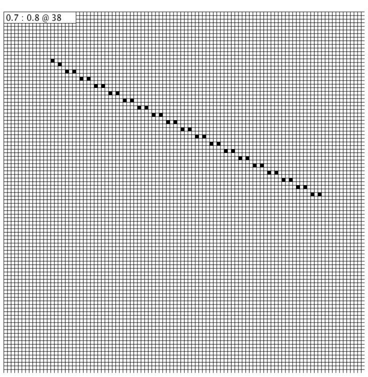
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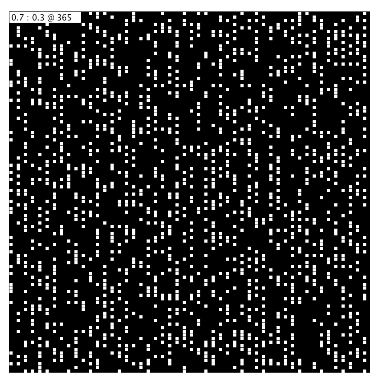
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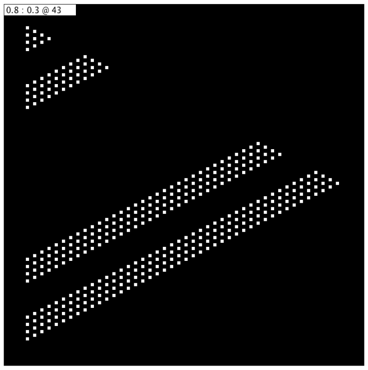
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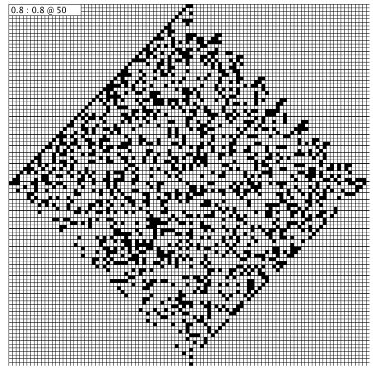
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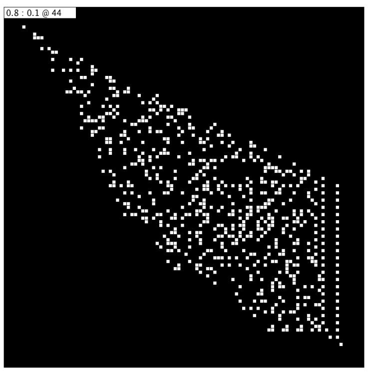
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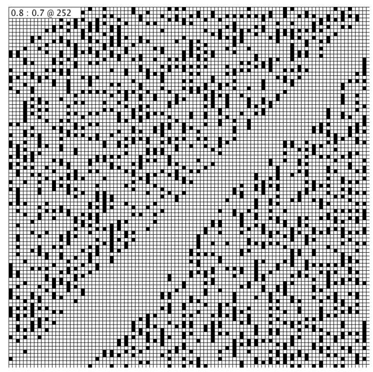
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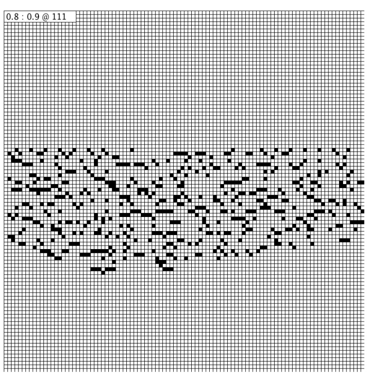
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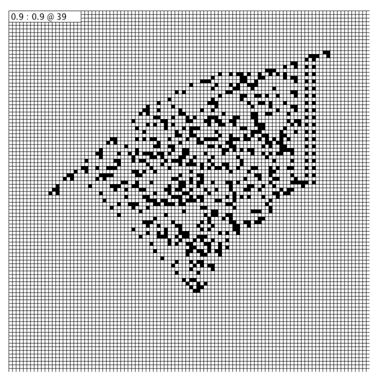
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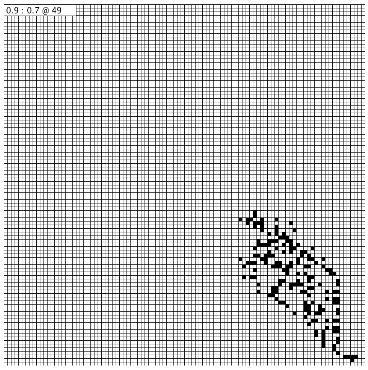
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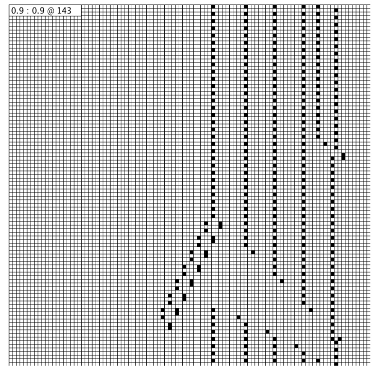
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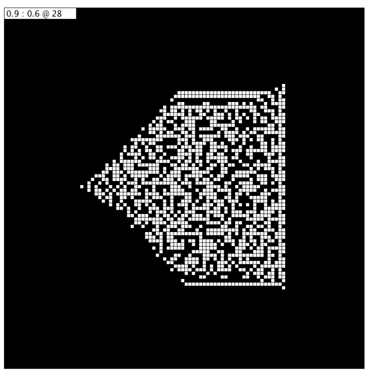
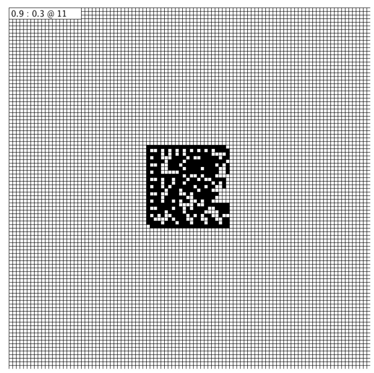

0.9 : 0.6 @ 28


0.9 : 0.3 @ 11

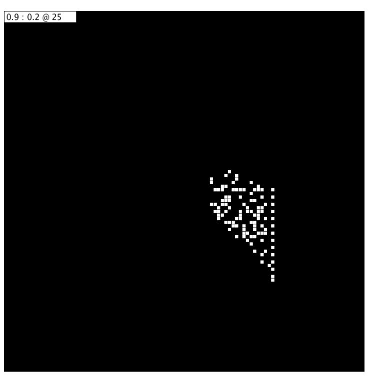

0.9 : 0.2 @ 25

Figure 1.1 : Number of Class 4 behavior recorded for each value of PC

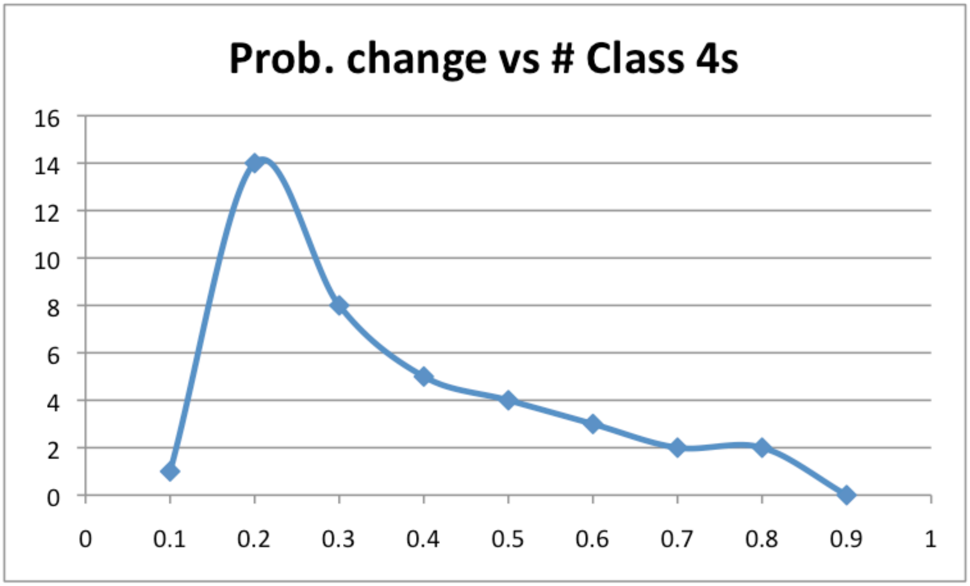


Figure 1.2: Number of Class 4 behaviors recorded for each value of CR

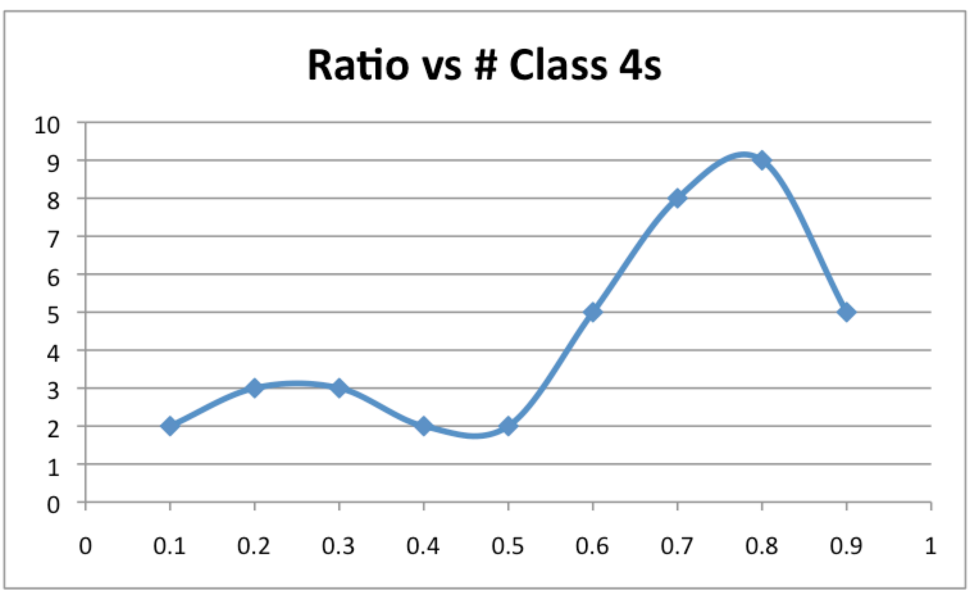


Figure 1.3: Distribution of class 4 behaviors in parameter space

					Probability of Change					
	E1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.1	1	1	2	1	2	4	2	3	2
	0.2	2	1	4	2	2	2	2	3	2
	0.3	1	2	3	2	3	3	3	3	3
	0.4	1	2	3	3	3	3	3	3	3
	0.5	1	2	3	3	3	3	3	3	3
	0.6	2	2	1	3	3	3	3	3	3
	0.7	1	4	3	3	3	3	3	3	3
	0.8	2	2	2	3	3	3	3	3	2
	0.9	4	1	2	1	1	1	1	2	1
	E2									
	0.1	1	1	2	3	3	2	1	3	3
	0.2	1	1	3	4	2	2	1	3	3
	0.3	1	2	3	1	2	3	3	2	3
	0.4	2	4	3	2	3	3	3	3	3
	0.5	2	1	3	3	3	3	3	3	3
	0.6	2	2	2	2	3	3	3	3	3
	0.7	2	4	4	3	3	4	3	2	3
	0.8	1	2	2	4	2	3	3	3	3
	0.9	2	2	2	2	2	1	3	2	3
	E3									
	0.1	1	2	2	2	4	2	3	2	2
	0.2	2	2	1	2	1	2	2	3	3
	0.3	2	1	3	3	3	3	3	3	2
	0.4	2	2	3	3	3	3	3	3	3
	0.5	2	2	2	3	3	3	3	3	3
	0.6	2	4	3	3	3	3	3	3	3
	0.7	2	2	2	3	2	3	3	3	3
	0.8	2	2	4	4	4	3	4	3	3
	0.9	2	2	2	2	3	2	3	4	1
Ratio	E4									
	0.1	1	1	1	2	2	2	2	2	2
	0.2	2	1	2	2	3	3	3	2	3
	0.3	2	2	1	1	3	3	3	3	3
	0.4	1	4	3	3	3	3	3	3	3
	0.5	2	4	3	3	3	3	3	3	3
	0.6	2	2	3	3	4	3	3	3	3
	0.7	2	4	2	3	3	3	3	3	3
	0.8	2	2	4	3	4	1	2	3	3
	0.9	1	2	1	1	2	1	2	2	3
	E5									
	0.1	1	1	2	2	1	1	1	2	2
	0.2	2	2	3	3	2	2	2	1	3
	0.3	1	4	3	3	2	3	2	3	3
	0.4	2	2	3	2	2	3	3	3	3
	0.5	2	2	3	3	3	3	3	3	3
	0.6	2	4	3	2	3	3	3	3	3
	0.7	2	2	3	3	3	3	3	3	3
	0.8	2	2	2	3	3	3	3	2	3
	0.9	2	4	1	4	2	4	1	2	2
	E6									
	0.1	2	2	2	2	2	2	3	3	2
	0.2	2	2	4	2	2	3	1	2	3
	0.3	2	4	2	3	3	3	3	3	3
	0.4	2	2	3	3	3	3	3	3	3
	0.5	2	2	3	3	3	3	3	3	3
	0.6	2	4	4	3	3	3	3	3	3
	0.7	2	4	3	3	3	3	3	3	3
	0.8	2	2	4	3	3	2	3	3	2
	0.9	2	2	2	2	3	3	1	2	2
	E7									
	0.1	1	1	2	2	2	2	2	1	2
	0.2	2	2	2	2	3	3	2	2	3
	0.3	2	2	3	3	3	3	4	3	3
	0.4	2	2	2	3	3	3	3	3	3
	0.5	2	4	3	3	2	3	3	3	3
	0.6	2	2	2	3	3	3	3	3	3
	0.7	2	2	1	4	3	3	3	4	3
	0.8	2	2	4	2	3	3	3	3	3
	0.9	2	2	2	2	3	3	2	3	1

DISCUSSION

Recall the central question of this paper.

"What types of laws will develop complexity in a binary-state system, from the simplest starting point"?

Consider Figure 1.1 above. The dataset comprising this experiment clearly suggests that the frequency of class 4 behaviors spiked dramatically when PC takes the value 0.2. PC decreases (almost exponentially) as PC increases from 0.2 finally reaching a zero frequency when PC is 0.9. The frequency of class 4 behavior when PC is at 0.1 is only 1. It is 14 when PC is 0.2, and 8 when PC is 0.3. Keep in mind that only 63 instances of PC 0.2 were observed so the result is really saying, '22% of observed systems with PC 0.2 exhibited Class 4 behavior'.

On my view this result provides a partial answer to the question above. In short, laws that define the probability of change in the system to be about 20% - but no lower – will tend to exhibit more complex behavior. Laws that define the probability of change to be greater than 20% will decrease in likelihood of exhibiting complex behavior as the distance from 20% grows – and laws where the probability of change in the system is lower than 20% drop off sharply in complexity.

Consider Figure 1.2 above. The trend it suggests is not so dramatic as with PC. However is seems clear that systems where the ratio of change types (recall off to on over on to off) is higher, exhibit more complex behavior, peaking at 0.8 after which a moderate drop off occurs. Note that only 63 instances of each parameter value were observed, so the data says of the CR 0.8 value, '14% of observed systems with CR 0.8 exhibited class 4 behavior'.

We would expect then that the main clustering of class 4 occurrences in Figure 1.3 above would be in the regions close to PC 0.2 and CR 0.8. This is indeed the case.

In the style of Langton, I suggest that the PC result indicates that complex behavior most frequently occurs on the 'edge' of something. On the edge of stasis, not on the edge of chaos. Systems that take PC 0.1 i.e. only change their state 10% of the time are by definition very static. Furthermore chaotic behavior only becomes widespread when systems take PC values of about 50% or more. Sadly, *the edge of stasis* doesn't sound quite as sexy as *the edge of chaos*.

I find it difficult to interpret the CR result. Before conducting the experiment I guessed that complementary values of CR (e.g. 0.1 and 0.9, 0.2 and 0.8 etc) would exhibit more or less the same behavior. This was based upon the notion that whether an element is turning off or on is somewhat arbitrary – what's important is that there is a change in state. Another way of putting this notion is that watching the evolution of these systems with the colors reversed (something I did do) does not affect the character of their behavior from the spectator's point of view at all. The CR result suggests that the strong bias is important.

Limitations

The rule-state space is quite huge – comprising some 2^512 rules. Clearly this presents a problem for anyone wishing to make generalizations about the entire domain. I only observed 567 distinct rules – and so can really only make very weak claims about the entire domain. The main obstacle standing in the way of doing really extensive explorations of the domain is the difficulty of programming a machine to recognize 'complex behavior' and then automating the experiment. For the time being humans need to observe the system and make a subjective decision. This in itself is a limitation. When I showed some of these systems to a friend we found ourselves disagreeing in a few instances on what was complex and what was not. I tried to adhere to Wolfram's classification schema though as I shall discuss, I think this schema has insufficiently many categories.

A class of systems where Wolfram's classification schema comes up short is the chaotic regimes (class 3). As is evidenced by the image-set chaotic regimes expand in very different ways. Some expand quickly as a geometric figure (box, diamond, triangle) others expand slowly and organically. Some start off as completely symmetrical non-chaotic regimes and then deteriorate into static when the local structures 'hit' each other. I think a richer classification schema is called for which is able to differentiate between these very different chaotic regimes. It is quite clear, even after such a limited number of experiments, that inside of non-critical regions (i.e. not PC 0.2 or CR 0.8), behavior exists on a continuum. Wolfram's scheme is similarly impoverished with regard to class 2 systems.

Where to Go From Here

I think this study supports the notion that there are regions of rule-space where complexity inducing rules cluster around certain rule-characteristics. Were I to do this study again, I would conduct a more fine-grained walk through rule space. Perhaps even a totally random walk through rule space – and attempt to characterize each complex system in terms of PC, CR and a few more parameters. Here are some candidate parameters for the future:

Informational Simplicity (IS): The number of statements a rule can minimally be expressed in. Clearly every rule can be expressed in 512 conditional statements. However we could reduce this by clustering antecedents. For example a rule might state, as does *CGOL* if the sum of neighboring cell's states is *x* do *y*, else...

Stasis Ratio (SR): The number of component transitions from on to on, divided by the number of component transitions from off to off.

Antecedent evenness (AE): The number of component transitions to on/off where the states of the antecedent sum to an even number, divided by the number of component transitions.

Antecedent oddness (AO): The number of component transitions to on/off where the states of the antecedent sum to an odd number, divided by the number of component transitions.

Antecedent primacy (AP): The number of component transitions to on/off where the states of the antecedent sum to a prime number, divided by the number of component transitions.

General Concluding Remarks

What might this study suggest about the physical world? If we accept Wheeler's notion of 'It from Bit,' then perhaps quite a bit (no pun intended). For example, what the PC result might suggest is that laws of physics that necessitate state changes about 20% of the time may be true of a universe which exhibit comparable levels of complexity to our own. Clearly this is a long shot. For one, the system examined here contains only a tiny number of elements – and does not expand, as we believe the universe to. Furthermore, it only exists in 2 spatial dimensions – and generally we believe our universe to exist in at least 3. It is possible, perhaps probable that critical values of PC and CR would be quite different for systems that more closely model the physical universe.

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